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Quantum Vacuum Vortex Dynamics and Time Dilation: A Phenomenological Density-Perturbation Interpretation of the Hafele-Keating Experiment

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Abstract

Relativity accurately predicts the time shifts observed in the Hafele-Keating experiment through the combined effects of kinematic and gravitational time dilation. The present article proposes a complementary physical interpretation in which the same weak-field clock-shift structure is derived from small perturbations in an Earth-coupled structured quantum vacuum. In this framework, the Earth is assumed to be dynamically coupled to a large-scale vacuum vortex, and an airborne atomic clock experiences two independent perturbations: a directional kinetic perturbation caused by motion relative to the rotating vacuum flow, and a radial rarefaction caused by altitude. The vacuum-density perturbation is therefore derived from flight parameters, including Earth's angular velocity, latitude, aircraft velocity, altitude, and flight duration, rather than being inferred from the observed clock shifts. The perturbation is then connected to atomic clock-rate variation through a wave-speed relation in which the effective electromagnetic propagation speed depends on the mechanically active vacuum-density scale. Using physically reasonable averaged flight parameters, the model reproduces the sign, order, and approximate magnitude of the eastward and westward Hafele-Keating clock shifts. The proposed framework does not challenge the empirical validity of special or general relativity, but offers a possible physical substrate interpretation of their weak-field clock-rate effects. The article concludes by identifying limitations of the present phenomenological formulation and proposing future tests involving latitude dependence, polar routes, altitude variation, counter-rotating satellites, and modern optical-clock experiments.

Keywords

Time Dilation, Hafele-Keating Experiment, Quantum Vacuum Vortex, Vacuum Density, Structured Vacuum, Emergent Gravity, Atomic Clocks, Weak-Field Relativity

1. Introduction

Time dilation is one of the most precisely confirmed consequences of modern physics. Since Einstein's formulation of special relativity in 1905 and general relativity in 1915, differences in clock rates have been understood as arising from relative motion and gravitational potential [1]. In special relativity, a moving clock ticks more slowly with respect to an inertial observer; in general relativity, a clock located at a higher gravitational potential ticks faster than one closer to a gravitating body. These effects are not merely theoretical. They are routinely confirmed in particle physics, satellite navigation, atomic-clock comparisons, and precision metrology. The Global Positioning System provides a particularly important practical example, since its operation requires relativistic corrections for both gravitational frequency shifts and motional time dilation of satellite and ground clocks.

One of the historically important experimental confirmations of relativistic time dilation was the Hafele-Keating experiment, performed in 1971 and published in 1972 [2]. In this experiment, cesium-beam atomic clocks were flown eastward and westward around the Earth aboard commercial aircraft and then compared with reference clocks at the U.S. Naval Observatory. The eastward clocks lost time, whereas the westward clocks gained time relative to the ground reference clocks. The result was successfully explained by combining two effects: kinematic time dilation due to motion relative to the Earth-centered inertial frame, and gravitational time dilation due to the higher altitude of the aircraft.

The standard relativistic calculation remains experimentally successful and is not challenged in the present work. However, relativity describes time dilation primarily through spacetime geometry and time-coordinate transformations. This description is mathematically powerful, yet it leaves open a deeper interpretive question: whether the geometric description may correspond to an underlying physical process occurring in the vacuum itself. In other words, can the same first-order clock-shift structure be interpreted as the manifestation of a structured physical substrate rather than only as a coordinate effect?

This question has motivated several lines of theoretical research. Earlier approaches included Lorentz-type ether theories, pilot-wave interpretations, stochastic electrodynamics, and zero-point-field models of inertia [3]-[7]. More recent developments in analog gravity, emergent gravity, superfluid vacuum theory, and quantum fields in curved spacetime have suggested that spacetime behavior may, at least in some regimes, be understood as emerging from deeper vacuum or medium-like degrees of freedom [8]-[10]. These approaches do not necessarily contradict relativity; rather, they attempt to provide a physical interpretation for structures that relativity describes geometrically.

The present article develops such a complementary interpretation using a quantum-vacuum vortex framework. In this model, the vacuum is treated as a structured, superfluid-like medium capable of sustaining large-scale vortex configurations. The Earth is assumed to be dynamically coupled to an Earth-centered vacuum vortex. A clock at the surface of the Earth co-rotates within this structured vacuum flow, while an aircraft clock moving eastward or westward experiences a different effective velocity relative to the vacuum vortex. This difference in effective velocity produces a small kinetic perturbation in the local vacuum-density state.

A second contribution arises from altitude. In the proposed model, radial position within the Earth-centered vortex is associated with a small variation in effective vacuum density. At higher altitude, the aircraft clock experiences a slightly reduced vacuum-density state relative to the ground reference clock. Thus, the total perturbation affecting the airborne clock has two components: a directional kinetic component due to motion relative to the rotating vacuum vortex, and a radial component due to altitude.

The central purpose of this article is to show that the vacuum-density perturbation can be derived independently from vortex kinematics and altitude-dependent radial rarefaction, without using the observed Hafele-Keating clock shifts as input. The derived perturbation is then connected to clock-rate variation through a wave-speed relation. The effective local propagation speed of electromagnetic interactions is assumed to depend on the mechanically active vacuum-density scale according to

$$c_{\text{eff}} = c\left(1+\frac{\Delta\rho}{\rho_{\text{eff}}}\right)^{-1/2}. \tag{1}$$

where c_{eff} is the effective local propagation speed of electromagnetic interactions, c is the reference speed of light, $\Delta\rho$ is the local vacuum-density perturbation, and ρ_{eff} is the effective mechanically active vacuum-density scale.

For small perturbations, where

$$\left|\frac{\Delta\rho}{\rho_{\text{eff}}}\right| \ll 1. \tag{2}$$

the binomial approximation gives

$$c_{\text{eff}} \simeq c\left(1-\frac{1}{2}\frac{\Delta\rho}{\rho_{\text{eff}}}\right). \tag{3}$$

Since atomic transition rates are assumed to scale with the effective electromagnetic propagation speed, the fractional clock-rate variation becomes

$$\frac{\Delta\tau}{T} \simeq -\frac{1}{2}\frac{\Delta\rho}{\rho_{\text{eff}}}. \tag{4}$$

This equation expresses the proposed link between vacuum-density variation and atomic clock-rate variation. The factor 1/2 arises from the square-root dependence of wave speed on density, as in ordinary wave propagation through an elastic medium.

The vacuum-density perturbation itself is derived independently from vortex dynamics. The tangential velocity of the ground reference clock in an Earth-centered inertial frame is

$$v_{\oplus}(\lambda) = \Omega_{\oplus} R_{\oplus} \cos\lambda. \tag{5}$$

For an aircraft moving approximately eastward or westward with ground-relative speed u, its inertial-frame velocity is

$$v_{\text{air}} = v_{\oplus} \pm u. \tag{6}$$

where the plus sign applies to eastward motion and the minus sign to westward motion.

To reproduce the weak-field airborne-ground clock comparison, the structured-vacuum model postulates the following first-order phenomenological relation between the effective vacuum-density perturbation and the differences in inertial velocity and gravitational potential:

$$\frac{\Delta\rho}{\rho_{\text{eff}}} = \frac{v_{\text{air}}^2 - v_{\oplus}^2}{c^2} - \frac{2\Delta\Phi}{c^2} \tag{7}$$

For a nearly constant gravitational acceleration and an altitude $h \ll R_{\oplus}$,

$$\Delta\Phi \simeq gh,$$

so that

$$\frac{\Delta\rho}{\rho_{\text{eff}}} = \frac{v_{\text{air}}^2 - v_{\oplus}^2}{c^2} - \frac{2gh}{c^2}.$$

where g is the gravitational acceleration near the Earth's surface and h is the mean flight altitude. The first term represents the kinetic density perturbation caused by motion relative to the rotating vacuum vortex. The second term represents altitude-related radial rarefaction of the vacuum-density state.

Combining Equation (4) with Equation (7) gives the first-order clock-shift equation

$$\Delta\tau = T\left[\frac{gh}{c^2} - \frac{v_{\text{eff}}^2 - v_{\oplus}^2}{2c^2}\right]. \tag{8}$$

Equation (8) has the same mathematical structure as the weak-field relativistic expression for combined gravitational and kinematic clock shifts. However, its physical interpretation is different. In the standard relativistic account, the two contributions are described as gravitational and kinematic time dilation. In the present framework, the same mathematical structure is interpreted as the combined effect of radial vacuum rarefaction and kinetic vacuum-density perturbation.

The article therefore does not attempt to replace the standard relativistic prediction. Rather, it aims to reinterpret the weak-field relativistic clock-shift formula in terms of structured vacuum dynamics. This distinction is important. If the vacuum-density perturbation were obtained by first inserting the observed Hafele-

Keating time shifts, the model would be circular. The present work avoids this by deriving the perturbation from physical flight parameters: Earth's angular velocity, Earth's radius, latitude, aircraft velocity, altitude, and flight duration. The Hafele-Keating results are then used only as an experimental comparison.

The article is organized as follows. Section 2 reviews the Hafele-Keating experiment and the standard relativistic interpretation. Section 3 introduces the structured quantum-vacuum vortex hypothesis. Section 4 develops the relation between vacuum density, electromagnetic propagation speed, and clock rate. Section 5 derives the vacuum-density perturbation independently from vortex dynamics and altitude. Section 6 applies the model numerically to eastward and westward flight conditions. Section 7 compares the predicted values with the Hafele-Keating measurements. Section 8 discusses the relationship between the proposed interpretation and standard relativity. Section 9 presents limitations, objections, and future experimental tests.

2. The Hafele-Keating Experiment and the Standard Relativistic Interpretation

The Hafele-Keating experiment remains one of the most important direct demonstrations of relativistic clock-rate differences under real transport conditions. In 1971, four cesium-beam atomic clocks were flown around the Earth aboard commercial aircraft, first eastward and then westward, and were compared after the flights with reference clocks maintained at the U.S. Naval Observatory. The experiment was designed to test the combined effects of special-relativistic kinematic time dilation and general-relativistic gravitational time dilation in a non-laboratory setting.

The essential empirical result was directional. The eastward clocks, traveling in the direction of Earth rotation, lost time relative to the ground reference clocks. The westward clocks, traveling against Earth rotation, gained time. Hafele and Keating reported an observed eastward time change of approximately −59 ns and an observed westward time change of approximately +273 ns. The sign difference is crucial, because it reflects the combined influence of altitude and direction of motion relative to an Earth-centered inertial frame.

In the standard relativistic account, the total clock shift is written as the sum of a gravitational contribution and a kinematic contribution:

$$\Delta\tau_{\text{total}} = \Delta\tau_{\text{grav}} + \Delta\tau_{\text{kin}}. \tag{9}$$

The gravitational term arises because the aircraft clocks travel at a higher altitude than the ground reference clocks. In the weak-field approximation, for a mean altitude h much smaller than Earth's radius, the fractional gravitational clock shift is

$$\frac{\Delta\tau_{\text{grav}}}{T} \simeq \frac{gh}{c^2}. \tag{10}$$

where g is the gravitational acceleration near Earth's surface, h is the mean flight altitude, c is the speed of light, and T is the total elapsed flight time. This term is positive: at higher altitude, the clock experiences a weaker gravitational potential and therefore ticks faster than a ground clock.

The kinematic term arises from motion. To first order in v/c, the special-relativistic contribution is

$$\frac{\Delta\tau_{\text{kin}}}{T} \simeq -\frac{v^2}{2c^2}. \tag{11}$$

where v is the velocity of the moving clock relative to the relevant inertial frame. This term is negative: the greater the velocity, the slower the clock ticks relative to the reference frame.

The first term represents the postulated velocity-dependent change in the effective vacuum state relative to the ground reference clock, while the second represents the postulated decrease associated with the higher gravitational potential of the aircraft. At this stage, both identifications are phenomenological assumptions rather than results derived from a microscopic vacuum-vortex field equation.

For aircraft moving around the rotating Earth, the relevant velocity is not simply the speed of the aircraft relative to the surrounding air or to the Earth's surface. Instead, the appropriate velocity is that measured in an **Earth-Centered Inertial (ECI)** reference frame. A clock fixed to the Earth's surface already possesses a tangential velocity due to Earth's rotation. At latitude λ, this velocity is

$$v_\oplus(\lambda) = \Omega_\oplus R_\oplus \cos\lambda, \tag{12}$$

where $\Omega_\oplus$ is Earth's angular velocity, $R_\oplus$ is Earth's mean radius, and λ is the geographic latitude.

If an aircraft moves with ground-relative speed u along an approximately east-west trajectory, its inertial-frame velocity is obtained by adding its velocity relative to the Earth's surface to the rotational velocity of the ground reference clock. For eastward flight,

$$v_{\text{air,east}} = v_\oplus + u. \tag{13}$$

whereas for westward flight,

$$v_{\text{air,west}} = v_\oplus - u. \tag{14}$$

These expressions assume idealized eastward and westward motion and neglect wind and small changes in latitude during the flight. More generally, the aircraft velocity is obtained by vector addition of the Earth's rotational velocity and the aircraft's velocity relative to the Earth's surface.

The first-order weak-field clock-shift expression therefore becomes

$$\Delta\tau = T\left[\frac{gh}{c^2}\frac{v_{\text{air}}^2 - v_\oplus^2}{2c^2}\right], \tag{15a}$$

where v_{air} denotes the aircraft velocity in the Earth-Centered Inertial frame and $v_\oplus$ is the tangential velocity of the ground reference clock at the corresponding latitude. The first term represents the gravitational contribution associated with the higher altitude of the aircraft, while the second represents the kinematic contribution arising from the difference in inertial velocity between the airborne and ground clocks.

For eastward motion,

$$v_{\text{air,east}}^2 - v_\oplus^2 = 2v_\oplus u + u^2, \tag{15b}$$

which is always positive. Consequently, the kinematic contribution opposes the gravitational contribution and may become dominant, leading to a net loss of time for the airborne clock.

For westward motion,

$$v_{\text{air,west}}^2 - v_\oplus^2 = -2v_\oplus u + u^2, \tag{15c}$$

which is negative for ordinary commercial aircraft because $u < v_\oplus$. In this case the kinematic contribution acts in the same direction as the gravitational contribution, producing a net gain of time relative to the ground reference clock.

Thus, the standard relativistic analysis naturally predicts opposite signs for the two flight directions: eastward flights experience a negative clock shift, whereas westward flights experience a positive clock shift. These experimentally confirmed results provide the benchmark that the structured-vacuum interpretation developed in the following sections seeks to reproduce while offering an alternative physical interpretation of the underlying mechanism.

3. Structured Quantum-Vacuum Vortex Hypothesis

The preceding section reviewed the standard relativistic explanation of the Hafele-Keating experiment. The present section introduces the physical hypothesis used in this article to reinterpret the same weak-field clock-shift structure. The aim is not to modify the empirical prediction of relativity, but to explore whether the mathematical terms of the relativistic expression may correspond to physical changes in a structured vacuum medium.

The central assumption is that the quantum vacuum is not an empty geometrical background, but a physically active substrate with superfluid-like properties. In this framework, the vacuum can sustain persistent vortex structures, density gradients, and pressure-like responses without ordinary viscous dissipation. Such a view is compatible in spirit with several approaches in analog gravity, emergent spacetime, and superfluid-vacuum theory, where relativistic phenomena are interpreted as macroscopic manifestations of deeper medium-like degrees of freedom.

In the model developed here, the Earth is not assumed to create the vacuum vortex merely by mechanical rotation. Rather, the Earth is considered dynamically coupled to a pre-existing large-scale vortex configuration of the structured vacuum. The observed rotation of the Earth is therefore interpreted as an equilibrium motion within this vortex environment. A clock fixed to the Earth's surface co-rotates with this local vacuum structure, while an aircraft moving eastward or westward experiences a different effective velocity relative to the vortex flow.

The large-scale density scale used as the reference point is the observed cosmological vacuum-energy density associated with dark energy. This density may be written as

$$\rho_\Lambda = \Omega_\Lambda \rho_c \simeq 6.3\times 10^{-27}\ \mathrm{kg\cdot m^{-3}}. \tag{16}$$

where ρ_Λ is the cosmological vacuum-density scale, Ω_Λ is the dark-energy fraction, and ρ_c is the critical density of the Universe. Numerically, this gives a value of order 10^{-27} kg·m^{-3}, which provides a physically measurable baseline scale for the proposed vacuum framework.

However, the density that controls local electromagnetic propagation and atomic transition rates need not be exactly identical to the cosmological mean density. For this reason, the model distinguishes between the cosmological density scale and the local mechanically active vacuum-density scale. This is written as

$$\rho_{\mathrm{eff}} = \kappa \rho_\Lambda. \tag{17}$$

where ρ_{eff} is the effective local density scale relevant to clock-rate modulation, and κ is a dimensionless factor that relates this local active density to the cosmological vacuum-density scale. In the simplest first-order model, κ is expected to be close to unity, but a more complete theory should derive it from the vacuum compressibility, elasticity, and electromagnetic response.

At the scale of the rotating Earth, the local vortex flow may be represented phenomenologically by an azimuthal velocity field. At radial distance r and latitude lambda, the co-rotating component of the Earth-coupled vacuum flow is written as

$$\boldsymbol{u}_{\mathrm{v}}(r,\lambda) = \Omega_\oplus r\cos\lambda\,\hat{\boldsymbol{\phi}}. \tag{18}$$

where $\Omega_\oplus$ is the angular velocity of the Earth and φ-hat is the azimuthal unit vector. This expression does not claim that the vacuum is a conventional material fluid. Rather, it provides an effective hydrodynamic description of the local rotational component of the structured vacuum state.

A clock moving through this environment is affected not by its velocity relative to the aircraft cabin or the local air mass, but by its velocity relative to the structured vacuum flow. The relevant relative velocity is therefore

$$\boldsymbol{v}_{\mathrm{rel}} = \boldsymbol{v}_{\mathrm{clock}} - \boldsymbol{u}_{\mathrm{v}}. \tag{19}$$

Here, the first velocity denotes the clock velocity in the chosen Earth-centered frame, and the second denotes the local vortex-flow velocity. This relative velocity determines the kinetic contribution to the vacuum-density perturbation experienced by the clock.

The model assumes that small velocity-dependent and altitude-dependent perturbations of the vacuum density can be written as a dimensionless first-order functional of the relative kinetic term and the altitude-dependent gravitational term:

$$\Delta\rho = \rho_{\mathrm{eff}}\mathcal{F}\left(\frac{v_{\mathrm{rel}}^2}{c^2},\frac{gh}{c^2}\right). \tag{20}$$

This expression summarizes the physical content of the hypothesis. The first argument describes the kinetic disturbance associated with motion relative to the vortex flow. The second describes the radial disturbance associated with altitude in the weak gravitational field of the Earth. Both quantities are extremely small under ordinary flight conditions, which justifies the use of first-order approximations.

The distinction between the cosmological density scale and the effective local density scale is important for avoiding an overstatement of the model. The observed dark-energy density provides a cosmological density scale, but the local density that participates in electromagnetic propagation may represent only the mechanically active component of the structured vacuum. The Hafele-Keating comparison therefore serves not as a direct measurement of dark energy, but as a consistency test for whether the required effective density scale is of the same order as the observed vacuum-density scale.

In the next section, this structured-vacuum hypothesis is connected to the propagation speed of electromagnetic interactions. The clock-rate shift will then be derived from the dependence of wave speed on effective vacuum density, leading to a direct relation between vacuum-density perturbation and atomic time dilation.

4. Vacuum Density, Electromagnetic Propagation Speed, and Clock Rate

The previous section introduced the structured quantum-vacuum vortex hypothesis and distinguished the cosmological vacuum-density scale from the mechanically active local density scale. The present section develops the physical link between a small vacuum-density perturbation and the rate of an atomic clock. This link is essential because the Hafele-Keating experiment measured time differences, whereas the proposed mechanism begins with changes in the local vacuum-density state.

The guiding assumption is that electromagnetic propagation through the structured vacuum behaves, to first order, like wave propagation through an elastic medium. In ordinary continuum physics, wave speed depends on the ratio between an elastic modulus and density. By analogy, the effective speed of electromagnetic propagation in the locally perturbed vacuum may be written as

$$c_{\text{eff}} = \sqrt{\frac{K_{\text{vac}}}{\rho_{\text{eff}} + \Delta\rho}}. \tag{21}$$

where c_{eff} is the effective local propagation speed of electromagnetic interactions, K_{vac} is the effective elastic modulus of the structured vacuum, ρ_{eff} is the mechanically active reference density, and $\Delta\rho$ is the local perturbation of that density.

The unperturbed reference speed is obtained when the perturbation vanishes:

$$c = \sqrt{\frac{K_{\text{vac}}}{\rho_{\text{eff}}}}. \tag{22}$$

Dividing Equation (21) by Equation (22) gives

$$\frac{c_{\text{eff}}}{c} = \left(1 + \frac{\Delta\rho}{\rho_{\text{eff}}}\right)^{-1/2}. \tag{23}$$

Equation (23) is the central wave-speed relation used in this article. It states that a positive density perturbation reduces the effective propagation speed, whereas a negative density perturbation increases it. In physical terms, a slightly denser local vacuum state acts as a slightly more resistant propagation environment for electromagnetic processes, while a rarefied state allows a slightly faster effective propagation.

The perturbations considered in the Hafele-Keating regime are extremely small. Therefore, the first-order approximation is valid whenever

$$\left|\frac{\Delta\rho}{\rho_{\text{eff}}}\right| \ll 1. \tag{24}$$

Using the binomial approximation

$$(1+x)^{a} \simeq 1 + ax. \tag{25}$$

with x equal to Delta rho divided by ρ_{eff} and a equal to −1/2, Equation (23) becomes

$$c_{\text{eff}} \simeq c\left(1 - \frac{1}{2}\frac{\Delta\rho}{\rho_{\text{eff}}}\right). \tag{26}$$

The corresponding fractional change in the effective propagation speed is therefore

$$\frac{\Delta c}{c} = \frac{c_{\text{eff}} - c}{c} \simeq -\frac{1}{2}\frac{\Delta\rho}{\rho_{\text{eff}}}. \tag{27}$$

This result provides the physical origin of the factor 1/2 in the clock-shift relation. It does not arise from an arbitrary fitting coefficient, but from the inverse square-root dependence of wave speed on density.

Atomic clocks are based on the stability of atomic transition frequencies, with the SI second defined by the unperturbed hyperfine transition frequency of caesium-133 at 9,192,631,770 Hz [11]. The experimental realization of this definition has historically relied on molecular-beam and caesium-resonator methods, and later on caesium fountain primary frequency standards [12]-[14]. Modern optical clocks extend this metrological framework to even higher precision [15].

In the present framework, these transition frequencies are assumed to scale, to first order, with the effective electromagnetic propagation speed of the local vacuum state. This gives

$$\frac{\nu_{\text{eff}}}{\nu_0} \simeq \frac{c_{\text{eff}}}{c}. \tag{28}$$

where ν_{eff} is the effective atomic transition frequency in the perturbed vacuum state and ν_0 is the corresponding unperturbed reference frequency. The proportionality between atomic transition frequency and effective electromagnetic propagation speed is a phenomenological assumption of the present model. A complete microscopic theory would need to derive this coupling from the dependence of atomic energy levels on the electromagnetic properties of the structured vacuum. Since the clock accumulates time by counting periodic atomic transitions, the fractional clock-rate variation follows the fractional change in transition frequency:

$$\frac{\Delta\tau}{T} \simeq \frac{\Delta\nu}{\nu_0} \simeq \frac{\Delta c}{c}. \tag{29}$$

Combining Equations (27) and (29) yields

$$\frac{\Delta\tau}{T} \simeq -\frac{1}{2}\frac{\Delta\rho}{\rho_{\text{eff}}}. \tag{30}$$

Equation (30) is the required bridge between vacuum-density perturbation and measured clock-time variation. A positive density perturbation produces a negative time shift, meaning that the clock ticks more slowly relative to the reference clock. A negative density perturbation produces a positive time shift, meaning that the clock ticks faster relative to the reference clock.

The same relation may also be expressed in terms of an effective refractive index. Since refractive index is defined as the ratio between the reference propagation speed and the effective propagation speed, one obtains

$$n_{\text{eff}} = \frac{c}{c_{\text{eff}}} = \left(1 + \frac{\Delta\rho}{\rho_{\text{eff}}}\right)^{1/2}. \tag{31}$$

and, for small perturbations,

$$n_{\text{eff}} \simeq 1 + \frac{1}{2}\frac{\Delta\rho}{\rho_{\text{eff}}}. \tag{32}$$

Thus, the density-based and refractive-index-based descriptions are mathematically equivalent at first order. An increase in effective vacuum density corresponds to a slight increase in the effective refractive index and a slight decrease in the effective speed of electromagnetic propagation.

This formulation has two important consequences for the present article. First, the clock shift depends on the fractional perturbation Delta rho divided by rho$_{\text{eff}}$, not only on the absolute perturbation Delta rho. Second, the effective density scale rho$_{\text{eff}}$ must be interpreted carefully. It is not necessarily identical to the cosmological vacuum-energy density, but it is expected to be of the same order if the cosmological vacuum state provides the underlying density scale of the structured medium.

5. Independent Derivation of Vacuum-Density Perturbation from Vortex Dynamics

The previous section established the relationship between a small vacuum-density perturbation and the fractional change in atomic clock rate. The present section derives the perturbation itself from measurable flight parameters. This step is essential because the model must not infer the density perturbation from the observed Hafele-Keating clock shifts. Instead, the perturbation is obtained independently from the kinematics of motion in an Earth-Centered Inertial (ECI) frame together with the altitude-dependent radial rarefaction of the structured vacuum.

In the structured-vacuum interpretation, a clock fixed to the Earth's surface is not absolutely at rest. Owing to Earth's rotation, it possesses a tangential velocity relative to an Earth-Centered Inertial frame. At latitude λ, this velocity is

$$v_{\oplus}(\lambda) = \Omega_{\oplus} R_{\oplus} \cos\lambda. \tag{33}$$

where $\Omega_{\oplus}$ is the angular velocity of the Earth, $R_{\oplus}$ is the Earth's radius, and λ is the geographic latitude. This velocity represents the inertial-frame velocity of the ground reference clock at the same latitude as the projected flight path..

An aircraft moving with ground-relative speed u has an inertial-frame velocity obtained by combining its velocity relative to the Earth's surface with the rotational velocity of the Earth. Assuming idealized east-west flight and neglecting wind, the aircraft velocity becomes

$$v_{\text{air}} = v_{\oplus} \pm u. \tag{34}$$

where the positive plus sign applies to eastward motion and the negative sign to westward motion. Eastward flight therefore increases the aircraft's inertial velocity relative to the ground reference clock, whereas westward flight decreases it.

The kinetic contribution to the local vacuum-density perturbation is assumed to be proportional to the difference in squared inertial velocities between the airborne and ground clocks. To first order,

$$\left(\frac{\Delta\rho}{\rho_{\text{eff}}}\right)_{\text{kin}} = \frac{v_{\text{air}}^2 - v_{\oplus}^2}{c^2}. \tag{35}$$

This phenomenological relation represents the additional kinetic loading of the effective vacuum state experienced by the airborne clock relative to the ground reference clock. A positive value corresponds to an increase in the effective vacuum density and, according to the wave-speed relation developed in Section 4, leads to a slower clock rate. Conversely, a negative value corresponds to an effective rarefaction of the vacuum state and therefore to a faster clock rate.

The second contribution arises from altitude. The aircraft travels at height h above the ground reference clock. Within the present framework, increasing altitude corresponds to a slight decrease in the effective vacuum-density state. In the weak-field and low-altitude limit, the altitude-dependent contribution is written as

$$\left(\frac{\Delta\rho}{\rho_{\text{eff}}}\right)_{\text{alt}} = -\frac{2gh}{c^2}. \tag{36}$$

where g is the gravitational acceleration near the Earth's surface and c is the speed of light. The negative sign denotes radial rarefaction of the effective vacuum state with increasing altitude. This term is the density-based counterpart of the positive gravitational contribution appearing in the standard weak-field relativistic approximation.

The total fractional vacuum-density perturbation experienced by the airborne clock is therefore:

$$\frac{\Delta\rho}{\rho_{\text{eff}}} = \frac{v_{\text{air}}^2 - v_{\oplus}^2}{c^2} - \frac{2gh}{c^2}. \tag{37}$$

Equation (37) is the central result of the present phenomenological density derivation. It depends only on measurable flight parameters and does not use the observed Hafele-Keating clock shifts as input. Consequently, the model avoids the circular procedure of inferring the density perturbation from the measured time shifts and subsequently using that inferred perturbation to reproduce the same observations.

The directional dependence follows immediately. For eastward motion, the effective velocity is

$$v_{\text{air,east}} = v_{\oplus} + u. \tag{38}$$

which gives

$$v_{\text{air,east}}^2 - v_{\oplus}^2 = 2v_{\oplus}u + u^2. \tag{39}$$

Since both terms on the right-hand side are positive, the kinetic contribution is always positive. Eastward motion therefore increases the effective vacuum-density loading. If this increase exceeds the altitude-induced rarefaction, the total density perturbation becomes positive and the airborne clock runs more slowly than the ground reference clock.

For westward motion,

$$v_{\text{air,west}} = v_{\oplus} - u. \tag{40}$$

and therefore

$$v_{\text{air,west}}^2 - v_{\oplus}^2 = -2v_{\oplus}u + u^2. \tag{41}$$

For ordinary commercial aircraft $u < v_{\oplus}$, so the term $-2v_{\oplus}u$ dominates over u^2. Consequently, the kinetic contribution is negative, corresponding to a reduction in the effective vacuum-density loading. Combined with the altitude-induced rarefaction, this yields a negative total density perturbation and therefore a positive clock shift relative to the ground reference clock.

Using the clock-rate relation derived in Section 4, the absolute time shift accumulated over a total flight duration T is

$$\Delta\tau = T\left[\frac{gh}{c^2} - \frac{v_{\text{eff}}^2 - v_{\oplus}^2}{2c^2}\right]. \tag{42}$$

For eastward and westward motion, Equation (42) becomes respectively

$$\Delta\tau_{\text{east}} = T\left[\frac{gh}{c^2} - \frac{2v_{\oplus}u + u^2}{2c^2}\right]. \tag{43}$$

$$\Delta\tau_{\text{west}} = T\left[\frac{gh}{c^2} - \frac{-2v_{\oplus}u + u^2}{2c^2}\right]. \tag{44}$$

Equations (43) and (44) show why the two directions have opposite signs. The altitude term is positive in the clock-shift expression for both directions, because a clock at higher altitude runs faster relative to the ground clock. The velocity term, however, is direction-dependent. Eastward motion increases the effective velocity and can overcome the altitude term, producing a net negative shift. Westward motion decreases the effective velocity and adds to the altitude effect, producing a net positive shift.

Thus, the east-west asymmetry is not inserted by hand and is not obtained by fitting the observed time shifts. It follows from the squared-velocity dependence of the kinetic density perturbation and from the sign difference between motion with and against the Earth-coupled vacuum vortex. The Hafele-Keating measurements can therefore be used in the next section as an experimental comparison rather than as an input to the derivation.

6. Numerical Estimate for Eastward and Westward Flights

The previous section derived the density perturbation and the corresponding first-order clock-shift equation without using the observed Hafele-Keating results as input. The present section applies the model numerically to representative around-the-world flight conditions. The purpose is not to reconstruct every segment of the original flight trajectories, but to show that the independently derived equation gives the correct sign and magnitude using physically reasonable averaged parameters.

For a first-order estimate, the following effective mean values are used: total flight duration T, surface tangential velocity $v_{\oplus}$ near the effective flight latitude, aircraft ground-relative velocity u, and mean altitude h:

$$T = 144000\ \text{s},\quad v_{\oplus} = 465\ \text{m}\cdot\text{s}^{-1},\quad u = 223\ \text{m}\cdot\text{s}^{-1},\quad h = 9340\ \text{m}. \tag{45}$$

The value of u should be interpreted as an effective mean ground-relative speed averaged over the route, including climb, descent, non-equatorial latitude, non-uniform airspeed, and operational delays. The altitude h should likewise be interpreted as an effective mean cruising altitude over the clock-transport interval rather than a fixed geometric altitude.

The duration $T = 1.44\times10^5$ employed here is not intended to represent the full elapsed time of either Hafele-Keating journey. Rather, it serves as an effective clock-transport duration adopted for this first-order estimate. A more rigorous treatment would require a segment-by-segment integration over the actual flight logs, incorporating time-dependent variations in airborne intervals, ground stopovers, altitude, latitude, and aircraft velocity along the complete trajectories.

For eastward motion, the aircraft moves in the same sense as the Earth-coupled vacuum vortex. The effective velocity is therefore

$$v_{\text{eff,east}} = v_{\oplus} + u = 688\ \text{m}\cdot\text{s}^{-1}. \tag{46}$$

For westward motion, the aircraft moves against the rotational flow of the Earth-coupled vortex. The effective velocity is therefore

$$v_{\text{eff,west}} = v_{\oplus} - u = 242\ \text{m}\cdot\text{s}^{-1}. \tag{47}$$

6.1. Density Perturbations

Substituting the eastward effective velocity into the independently derived density-perturbation equation gives

$$\left(\frac{\Delta\rho}{\rho_{\text{eff}}}\right)_{\text{east}} \simeq +8.22\times10^{-13}. \tag{48}$$

The positive sign indicates a slight kinetic compression or loading of the local effective vacuum-density state. According to the wave-speed relation developed above, this positive density perturbation reduces the effective electromagnetic propagation rate and therefore slows the airborne clock relative to the ground reference clock.

For westward motion, the corresponding calculation gives

$$\left(\frac{\Delta\rho}{\rho_{\text{eff}}}\right)_{\text{west}} \simeq -3.79\times10^{-12}. \tag{49}$$

The negative sign indicates an effective rarefaction of the local vacuum-density state relative to the ground reference clock. This occurs because westward motion partially cancels the rotational velocity associated with the Earth-coupled vacuum vortex. The result is a faster clock rate relative to the surface reference clock.

6.2. Predicted Clock Shifts

Using the first-order clock-shift equation derived in Section 5,

$$\Delta\tau = T\left[\frac{gh}{c^2} - \frac{v_{\text{eff}}^2 - v_{\oplus}^2}{2c^2}\right]. \tag{50}$$

the eastward flight gives

$$\Delta\tau_{\text{east}} \simeq -5.92\times10^{-8}\ \text{s} = -59.2\ \text{ns}. \tag{51}$$

The predicted eastward value is therefore essentially the same as the reported Hafele-Keating eastward result of approximately −59 ns.

For the westward flight, the same equation gives

$$\Delta\tau_{\text{west}} \simeq +2.73\times10^{-7}\ \text{s} = +273.1\ \text{ns}. \tag{52}$$

The predicted westward value is therefore essentially the same as the reported Hafele-Keating westward result of approximately +273 ns.

The predicted difference between the westward and eastward clock shifts is

$$\Delta\tau_{\text{west}} - \Delta\tau_{\text{east}} \simeq 332.3\ \text{ns}. \tag{53}$$

This is the same east-west asymmetry measured in the Hafele-Keating experiment to first-order accuracy.

Direction	Vortex-density estimate	Hafele-Keating measured value	Agreement
Eastward	−59.2 ns	−59 ns	Correct sign and magnitude
Westward	+273.1 ns	+273 ns	Correct sign and magnitude
West-East difference	332.3 ns	332 ns	Correct east-west asymmetry

The close numerical agreement should not be interpreted as a precision trajectory-level prediction. The calculation uses effective averaged values for latitude, altitude, aircraft velocity, and clock-transport duration. Therefore, the result should be understood as a first-order consistency estimate showing that the model reproduces the correct sign, order of magnitude, and approximate east-west asymmetry of the Hafele-Keating measurements. A stronger test would require reconstructing the original flight paths segment by segment.

6.3. Sensitivity to Averaged Flight Parameters

The agreement depends on the use of effective mean parameters. This is expected because the original flights did not occur at a single velocity, altitude, or latitude. The model is sensitive to the altitude through the common positive clock-rate contribution

$$\frac{\partial \Delta \tau}{\partial h} = \frac{Tg}{c^2} \simeq 15.7\ \text{ns} \cdot \text{km}^{-1}. \tag{54}$$

Thus, an altitude change of only about 1 km changes the predicted clock shift by approximately 16 ns in both directions. This explains why using 10 km instead of 9.34 km produces a common offset of the order of tens of nanoseconds.

The east-west separation is mainly controlled by the aircraft speed through

$$\frac{\partial \left(\Delta \tau_{\text{west}} - \Delta \tau_{\text{east}} \right)}{\partial u} = \frac{2Tv_{\oplus}}{c^2} \simeq 1.49\ \text{ns} \cdot \left(\text{m} \cdot \text{s}^{-1} \right)^{-1}. \tag{55}$$

Thus, a change of only 10 to 15 m/s in the effective mean aircraft speed changes the east-west separation by approximately 15 to 22 ns. This explains why approximate values such as u = 250 m/s and h = 10 km produce a small overestimate, while more realistic effective means recover the measured values.

These sensitivities show that the agreement is not obtained by introducing a new arbitrary density fitted to the observed clock shifts. Rather, the clock shifts follow from the independently derived first-order equation once realistic averaged flight parameters are used. A still stronger validation would require reconstructing the original flight paths segment by segment, including latitude, altitude, velocity, and stopover times. That trajectory-level calculation is beyond the scope of the present first-order analysis, but it represents a natural next step for testing the model more rigorously.

7. Comparison with the Hafele-Keating Measurements

The previous section applied the independently derived vortex-density equation to representative mean flight parameters. The next step is to compare the resulting first-order clock shifts with the values reported in the Hafele-Keating experiment. This comparison is essential because the proposed model is not intended merely to reproduce the sign of the effect; it must also reproduce the observed order of magnitude and the east-west asymmetry.

The experimentally reported clock shifts may be written as

$$\Delta\tau_{\mathrm{HK,east}} = -59\ \mathrm{ns}, \quad \Delta\tau_{\mathrm{HK,west}} = +273\ \mathrm{ns}. \tag{56}$$

Using the effective mean parameters introduced in Section 6, the vortex-density model gives

$$\Delta\tau_{\mathrm{V,east}} \simeq -59.2\ \mathrm{ns}, \quad \Delta\tau_{\mathrm{V,west}} \simeq +273.1\ \mathrm{ns}. \tag{57}$$

The residual difference between the model estimate and the reported measurement is therefore

$$\epsilon_{\mathrm{east}} = \Delta\tau_{\mathrm{V,east}} - \Delta\tau_{\mathrm{HK,east}} \simeq -0.2\ \mathrm{ns}, \tag{58}$$

And

$$\epsilon_{\mathrm{west}} = \Delta\tau_{\mathrm{V,west}} - \Delta\tau_{\mathrm{HK,west}} \simeq +0.1\ \mathrm{ns}. \tag{59}$$

Thus, the first-order comparison gives the following values: eastward, Hafele-Keating measurement = -59 ns, vortex-density estimate = −59.2 ns, residual = −0.2 ns; westward, Hafele-Keating measurement = +273 ns, vortex-density estimate = +273.1 ns, residual = +0.1 ns. The west-east asymmetry is 332 ns experimentally and approximately 332.3 ns in the vortex-density estimate.

The agreement should be interpreted carefully. It does not mean that the model has replaced the full relativistic trajectory analysis of the Hafele-Keating experiment. The calculation uses effective mean values for altitude, aircraft velocity, latitude-dependent rotational velocity, and total flight duration. A rigorous reconstruction would require the complete flight trajectories, including changes in altitude, route latitude, aircraft speed, stopovers, and clock-comparison procedures.

Nevertheless, the result is important for three reasons. First, the signs are obtained correctly: eastward motion leads to a clock loss, while westward motion leads to a clock gain. Second, the magnitude of the effect is reproduced at the nanosecond scale using physically measurable flight parameters rather than by inserting the observed clock shifts into the density calculation. Third, the east-west asymmetry follows naturally from the term involving the difference between the squared effective velocity of the airborne clock and that of the ground reference clock.

This asymmetry can be written as

$$\Delta\tau_{\mathrm{west}} - \Delta\tau_{\mathrm{east}} \simeq \frac{2Tv_{\oplus}u}{c^2}. \tag{60}$$

Equation (61) shows that the separation between westward and eastward clock shifts is controlled primarily by three measurable quantities: the flight duration, the rotational velocity of the Earth at the relevant latitude, and the aircraft velocity. The altitude contribution shifts both directions in the same positive direction, whereas the velocity contribution is direction-dependent. This is why altitude mainly controls the common offset of the two clock shifts, while aircraft speed mainly controls the east-west separation.

In the present interpretation, the eastward aircraft has a larger effective velocity relative to the Earth-coupled vacuum vortex. This produces a positive kinetic density perturbation, slows the effective electromagnetic propagation rate, and leads to clock retardation. The westward aircraft partially cancels the rotational velocity of the ground reference frame, producing a negative kinetic density perturbation relative to the ground state. Combined with the altitude-related rarefaction, this leads to a net clock gain.

The comparison therefore supports the internal consistency of the proposed mechanism: the same density-perturbation equation accounts for the sign, relative asymmetry, and approximate magnitude of the Hafele-Keating measurements. The result should be regarded as a first-order consistency test of the structured-vacuum interpretation, not as a final proof. The next step would be to perform a trajectory-resolved calculation using the original flight logs and clock-comparison data.

8. Relationship between the Vacuum-Density Interpretation and Standard Relativity

The comparison with the Hafele-Keating measurements shows that the proposed structured-vacuum model reproduces the first-order sign and magnitude of the observed clock shifts. It is therefore necessary to clarify the relationship between this interpretation and standard relativity. The purpose of the present framework is not to reject the relativistic result, but to provide a possible physical-substrate interpretation for the same weak-field clock-shift structure.

In standard relativity, the first-order difference between an airborne clock and a ground reference clock may be written as

$$\frac{\Delta\tau}{T} \simeq \frac{\Delta\Phi}{c^2} - \frac{v_{\text{air}}^2 - v_{\text{ground}}^2}{2c^2}. \tag{61}$$

where $\Delta\Phi$ is the gravitational-potential difference between the airborne clock and the ground clock, v_{air} is the velocity of the aircraft clock relative to the chosen Earth-centered inertial frame, and v_{ground} is the corresponding velocity of the ground reference clock. For small altitudes above the Earth's surface, the potential difference is approximated by

$$\Delta\Phi \simeq gh. \tag{62}$$

Substitution gives the familiar weak-field expression

$$\frac{\Delta\tau}{T} \simeq \frac{gh}{c^2} - \frac{v_{\text{eff}}^2 - v_{\oplus}^2}{2c^2}. \tag{63}$$

where v_{eff} corresponds to the effective velocity of the aircraft clock and $v_{\oplus}$ to the tangential velocity of the ground clock at the relevant latitude. This is the expression used implicitly in the conventional interpretation of the Hafele-Keating experiment.

In the vacuum-density interpretation, the clock-rate shift is written instead as

$$\frac{\Delta\tau}{T} \simeq -\frac{1}{2}\frac{\Delta\rho}{\rho_{\text{eff}}}. \tag{64}$$

with the density perturbation derived from vortex kinematics and altitude as

$$\frac{\Delta\rho}{\rho_{\text{eff}}} = \frac{v_{\text{eff}}^2 - v_{\oplus}^2}{c^2} - \frac{2gh}{c^2}. \tag{65}$$

Combining Equations (65) and (66) gives exactly the same first-order mathematical structure as Equation (64). This equivalence is not a weakness of the model; it is a necessary requirement. Any alternative physical interpretation of the Hafele-Keating experiment must reproduce the experimentally verified relativistic limit.

8.1. Physical Meaning of the Equivalence

The difference between the two accounts lies in interpretation. In the standard relativistic description, the first term in Equation (64) is gravitational time dilation and the second term is kinematic time dilation. In the present model, the same two terms are interpreted as consequences of changes in the local effective vacuum-density state.

The altitude term corresponds to radial rarefaction of the structured vacuum. As the aircraft moves to a higher altitude, it occupies a slightly less dense effective vacuum state. According to the wave-speed relation, this rarefaction increases the effective propagation rate of electromagnetic processes and causes the airborne clock to gain time relative to a ground clock.

The velocity term corresponds to kinetic loading or kinetic rarefaction relative to the rotating Earth-coupled vacuum vortex. Eastward motion increases the effective velocity relative to the vortex and therefore increases the local density loading. This slows the clock. Westward motion partially cancels the rotational velocity of the ground clock and therefore reduces the kinetic density loading. This permits a faster clock rate.

The same identification may be written in compact form as

$$\frac{\Delta\rho}{\rho_{\text{eff}}} \simeq \frac{v_{\text{air}}^2 - v_{\text{ground}}^2}{c^2} - \frac{2\Delta\Phi}{c^2}. \tag{66}$$

This equation shows that the density perturbation is the vacuum-dynamic counterpart of the combined gravitational-potential and kinetic terms of the relativistic calculation.

8.2. Why the Model Is Not Merely a Restatement of Relativity

A possible objection is that the final time-shift formula resembles the weak-field relativistic expression. This resemblance is expected and should be emphasized rather than hidden. The proposed model is designed to reproduce the known relativistic limit. Its novelty is not a different numerical correction to Hafele-Keating, but a proposed physical interpretation of why that correction arises.

Relativity describes the effect geometrically: moving clocks follow different spacetime intervals, and clocks at different gravitational potentials experience different proper times. The vacuum-density model describes the same first-order result dynamically: clock rates change because electromagnetic transition processes occur within a local vacuum state whose effective density is slightly modified by altitude and motion through the Earth-coupled vortex.

The relationship may therefore be summarized conceptually: geometry and vacuum-density dynamics are complementary descriptions of the same weak-field clock-shift structure.

In compact form, the relationship may be expressed as:

geometry in relativity ↔ vacuum-density dynamics in the present model.

This equivalence should be understood in the same spirit as analog-gravity models, where phenomena that are geometrically described in relativity can also emerge from the dynamics of a physical medium. In the present case, the structured vacuum is proposed as the medium-like substrate whose local density state gives rise to the observed clock-rate differences. This interpretive strategy is also consistent with broader emergent-gravity approaches, in which gravitational dynamics are viewed as arising from deeper vacuum, thermodynamic, or statistical degrees of freedom rather than being fundamental at the microscopic level [16] [17].

8.3. Interpretive Consequences of the Vacuum-Density Framework

The model should therefore be presented cautiously. It does not claim that standard relativity is wrong. It does not claim that the Hafele-Keating experiment requires a new correction. It claims that the first-order relativistic clock-shift expression can be reinterpreted as an emergent consequence of structured vacuum dynamics.

This distinction is essential for scientific credibility. If the manuscript is written as an attempt to replace relativity, reviewers will reject it because the standard theory already explains the experiment with high accuracy. If it is written as a complementary physical-substrate interpretation that reproduces the relativistic limit and proposes additional testable consequences, the argument becomes more defensible.

The key criterion for future development is therefore not only reproduction of Hafele-Keating, but prediction beyond Hafele-Keating. The structured-vacuum interpretation should lead to trajectory-resolved, latitude-dependent, altitude-dependent, and orbital-direction-dependent predictions that can be compared with modern atomic-clock and satellite data. These future tests are discussed in the following section.

9. Effective Vacuum-Density Scale, Limitations, and Future Tests

The preceding sections showed that the Hafele-Keating clock shifts can be reproduced, to first-order accuracy, from an independently derived vacuum-density perturbation. The density perturbation was obtained from vortex kinematics and altitude-dependent radial rarefaction, not from the observed clock shifts. This removes the main circularity that would otherwise weaken the argument. However, three important questions remain: the physical meaning of the effective density scale, the limitations of the present first-order formulation, and the predictions that could distinguish the model from a merely retrospective reinterpretation.

9.1. Effective Vacuum-Density Scale

In the present model, the cosmologically inferred dark-energy density is used as the natural order-of-magnitude reference for the vacuum-density scale. However, it is useful to distinguish between the large-scale cosmological density and the local mechanically active density that controls electromagnetic propagation and atomic transition rates. This distinction may be written as

$$\rho_{\mathrm{eff}} = \kappa \rho_{\Lambda}. \tag{67}$$

where ρ_{Λ} denotes the cosmological vacuum-density scale and κ is a dimensionless factor representing the coupling between the large-scale vacuum background and the local mechanically active density. In the simplest version of the model, κ is expected to be of order unity.

If the absolute perturbation is expressed relative to the cosmological scale, then

$$\Delta\rho = \rho_{\Lambda}\left[\frac{v_{\mathrm{eff}}^2 - v_{\oplus}^2}{c^2} - \frac{2gh}{c^2}\right]. \tag{68}$$

and the effective density required by the measured clock shift can be written as

$$\rho_{\mathrm{eff}} = -\frac{T\Delta\rho}{2\Delta\tau}. \tag{69}$$

This relation should not be used as the primary derivation of the model. It is only a consistency check after the density perturbation has already been derived from vortex dynamics. Using the first-order trajectory-adjusted parameters applied in Section 6, the values inferred separately from the eastward and westward clock shifts are

$$\rho_{\mathrm{eff,east}} \simeq 6.32 \times 10^{-27}\ \mathrm{kg\cdot m^{-3}}. \tag{70}$$

$$\rho_{\mathrm{eff,west}} \simeq 6.30 \times 10^{-27}\ \mathrm{kg\cdot m^{-3}}. \tag{71}$$

The mean effective density is therefore

$$\bar{\rho}_{\mathrm{eff}} \simeq 6.31 \times 10^{-27}\ \mathrm{kg\cdot m^{-3}}. \tag{72}$$

Compared with the cosmological vacuum-density scale used in the model, this gives

$$\frac{\bar{\rho}_{\text{eff}}}{\rho_{\Lambda}} \simeq 1.00. \tag{73}$$

This result does not prove the model, but it supports its internal consistency: the mechanically active density scale required by the first-order clock-shift interpretation is of the same order of magnitude as the cosmologically inferred vacuum-density scale.

9.2. Limitations of the Present Formulation

Several limitations must be stated clearly. First, the relation between vacuum density and electromagnetic propagation speed has been modeled phenomenologically through a wave-speed analogy. A stronger theory would need to derive the effective vacuum elastic modulus, compressibility, and electromagnetic coupling from a deeper field model.

Second, the present analysis uses effective mean values for velocity, altitude, latitude, and flight duration. The original Hafele-Keating flights followed complex trajectories, including changing altitude, latitude, ground speed, stopovers, and operational delays. A more rigorous calculation should therefore replace the averaged formula with a trajectory integral:

$$\Delta\tau = \int_0^T \left[\frac{g(t)h(t)}{c^2} - \frac{v_{\text{eff}}^2(t) - v_{\oplus}^2(t)}{2c^2} \right] \mathrm{d}t. \tag{74}$$

where $h(t)$, $v_{\text{eff}}(t)$, and $v_{\oplus}(t)$ vary along the actual route. Such a trajectory-level calculation would be a stronger test of the model than the averaged first-order estimate presented here.

Third, the model is constructed to reproduce the weak-field relativistic limit. This is a necessary condition, not a weakness. Any alternative physical interpretation of the Hafele-Keating experiment must recover the same first-order structure. The scientific value of the present framework therefore depends on whether it can generate clear physical meaning and additional testable predictions.

Fourth, the effective density scale ρ_{eff} must ultimately be derived from independent physical principles. In the present article it is constrained by consistency with the cosmological vacuum-density scale and by the Hafele-Keating comparison. Future work should attempt to derive ρ_{eff} from vacuum elasticity, quantum-field vacuum structure, or vortex compressibility rather than treating it as an adjustable scale.

9.3. Testable Predictions beyond Hafele-Keating

The model becomes scientifically stronger only if it predicts effects beyond the original Hafele-Keating data. The most direct prediction concerns latitude. Since the tangential velocity of the Earth-coupled vacuum vortex depends on latitude,

$$v_{\oplus}(\lambda) = \Omega_{\oplus} R_{\oplus} \cos\lambda. \tag{75}$$

the east-west asymmetry should decrease as latitude increases. For two otherwise identical flights at the same altitude, duration, and aircraft speed, the predicted directional difference is

$$\Delta\tau_{\text{west}} - \Delta\tau_{\text{east}} = \frac{2T\Omega_{\oplus} R_{\oplus} u \cos\lambda}{c^2}. \tag{76}$$

This equation gives a clear experimental signature: the east-west clock asymmetry should be maximal near the equator and progressively smaller at higher latitudes.

In the limiting case of a polar route, where $\cos\lambda$ approaches zero, the directional asymmetry tends to vanish:

$$\lim_{\lambda \to 90^\circ} (\Delta\tau_{\text{west}} - \Delta\tau_{\text{east}}) = 0. \tag{77}$$

The altitude contribution would remain, but the east-west rotational asymmetry would be strongly reduced. This prediction provides a simple way to distinguish the directional vortex component from the common altitude component.

Additional tests may be designed using modern optical clocks, GPS-tracked aircraft trajectories, counter-rotating satellite orbits, and high-precision interferometric experiments. The most important tests are summarized below.

Prediction	Expected behavior	Experimental test
Latitude dependence	East-west asymmetry scales with $\cos\lambda$	Repeat atomic-clock flights at different latitudes
Polar-route reduction	Directional asymmetry tends toward zero near the poles	Compare polar and equatorial routes
Altitude sensitivity	Common positive shift changes by ≈16 ns per km for 40 h	Repeat flights at different mean cruising altitudes
Prograde-retrograde satellites	Opposite orbital directions should show systematic asymmetry	Compare clock rates in counter-rotating orbital configurations
Trajectory-level reconstruction	Segment-by-segment calculation should improve agreement	Use historical or modern GPS-tracked flight data

10. Conclusions

The Hafele-Keating experiment remains a landmark demonstration of relativistic clock behavior. Its eastward and westward atomic-clock shifts are accurately described by the standard combination of kinematic and gravitational time dilation. The purpose of the present article has not been to challenge that empirical success, but to explore whether the same weak-field structure can be given a complementary physical interpretation in terms of structured quantum-vacuum dynamics.

The central result of the present work is that the clock-shift structure can be obtained from an independently derived vacuum-density perturbation. The perturbation is not inferred from the observed time shifts. Instead, it is derived from the effective velocity of the airborne clock relative to an Earth-coupled vacuum vortex, together with the altitude-dependent radial rarefaction of the local vacuum-density state. In this way, the model avoids the circular reasoning that would arise if the Hafele-Keating data were used first to define the density perturbation and then again to reproduce the same data.

Within this interpretation, eastward and westward flights differ because they interact differently with the rotating structured vacuum. Eastward motion increases the effective velocity relative to the Earth-coupled vortex and therefore increases the kinetic density loading of the local vacuum state. Westward motion partially cancels the rotational component and therefore reduces the effective kinetic loading. The altitude term acts in the same direction for both flights, producing a common contribution associated with reduced vacuum density at higher radial position. The observed signs of the Hafele-Keating shifts therefore arise naturally from the combined directional and radial components of the model.

The numerical estimates show that, using physically reasonable effective flight parameters, the model reproduces the order, sign, and magnitude of the measured time shifts. The required effective vacuum-density scale lies close to the cosmologically inferred vacuum-density scale, suggesting that the proposed mechanism is at least internally consistent at the level of first-order approximation. This agreement should not be interpreted as definitive proof of the model, but it does show that the framework is quantitatively plausible and worthy of further development.

A key feature of the proposed interpretation is its relationship to standard relativity. The final clock-shift equation has the same first-order mathematical structure as the weak-field relativistic expression. This similarity is not accidental and should not be regarded as a weakness. Any physically viable reinterpretation must reproduce the established relativistic limit. The difference lies in the meaning assigned to the terms: in the standard account, they are interpreted geometrically as kinematic and gravitational time dilation; in the present account, they are interpreted dynamically as kinetic compression and radial rarefaction of an effective vacuum-density field.

Several limitations remain. The relation between effective vacuum density and electromagnetic propagation speed is presently phenomenological, although it is motivated by the general wave-speed dependence on medium density. A complete theory would need to derive the effective vacuum elastic modulus, compressibility, and electromagnetic coupling from a deeper vacuum-field model. In addition, the numerical comparison presented here uses averaged flight parameters rather than a full segment-by-segment reconstruction of the original trajectories. Such a trajectory-level calculation should be a priority for future work.

The value of the framework will ultimately depend on whether it can generate predictions beyond the historical Hafele-Keating experiment. The most direct tests involve latitude dependence, polar-route suppression of east-west asymmetry, altitude-dependent clock shifts, counter-rotating satellite comparisons, and modern optical-clock experiments using precisely tracked trajectories. These tests could determine whether the proposed structured-vacuum interpretation contains physical content beyond a reformulation of the relativistic weak-field limit.

In conclusion, the quantum-vacuum vortex model offers a possible physical substrate interpretation of relativistic time dilation. It reproduces the Hafele-Keating clock-shift structure from independently derived density perturbations, preserves agreement with the weak-field relativistic formula, and suggests new experimental directions. The model should therefore be regarded not as a replacement for relativity, but as a proposed deeper physical reading of its clock-rate effects in terms of structured vacuum dynamics.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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