Scientific
Research
Publishing

A New Theory on the Origin and Nature of the Fine Structure Constant

Nader Butto

Dgania, Petah Tikva, Israel
Email: nader.butto@gmail.com

Abstract

The nature and the origin of the fine structure are described. Based on the vortex model and hydrodynamics, a comprehensible interpretation of the fine structure constant is developed. The vacuum considered to have superfluid characteristics and elementary particles such as the electron and Hydrogen molecule are irrotational vortices of this superfluid. In such a vortex, the angular rotation ω is maintained, and the larger the radius, the slower the rotational speed. The fine structure value is derived from the ratio of the rotational speed of the boundaries of the vortex to the speed of the vortex eye in its center. Since the angular rotation is constant, the same value was derived from the ratio between the radius of the constant vortex core and the radius of the hall vortex. Therefore, the constancy of alpha is an expression of the constancy relation in the vortex structure.

1. Introduction

The fine structure constant (α), also known as Sommerfeld's constant, was discovered to be a ubiquitous constant and is one of the fundamental constants in nature [1], characterizing the whole range of physics from elementary particles to atomic, mesoscopic, and macroscopic systems (similar to the speed of light, Planck's constant, and Newton's gravitational constant "G"). The values of these constants of nature determine the nature of our universe. A small difference (as little as 4%) in the value of the fine structure constant would have prevented stars from sustaining the nuclear reactions in their cores that produced carbon and allowed carbon-based lifeforms in our universe; for example, if α were

greater than 0.1, stellar fusion would be impossible, and no place in the universe would have been warm enough for survival [2].

The fine structure describes the splitting of the spectral lines of atoms caused by the interaction between the spin and orbital angular momenta of the outermost electron. It was first measured for the Hydrogen atom by Albert A. Michelson and Edward W. Morley in 1887 [3]. To explain the observed splitting or fine structure of the energy levels of the Hydrogen atom, Sommerfeld extended the Bohr theory to include elliptical orbits and the relativistic dependence of mass on velocity, deriving the Bohr-Sommerfeld model [4] [5]. Introduced into physics in 1916, the fine structure constant [6], has been discussed for decades. It is commonly denoted by α and is a unitless and dimensionless physical constant [7]. It is widely accepted that the value of α is related to the electromagnetic force between subatomic charged particles and determines how an atom holds its electrons. Thus, it is related to the elementary charge e, which characterizes the strength of the coupling of an elementary charged particle with the electromagnetic field:

$$\alpha = e^2 \big/ 4\pi \varepsilon_0 \hbar c$$

where e is the unit electromagnetic charge, ε_0 is the permittivity constant, $\hbar$ is Planck's constant divided by 2π, and c is the velocity of light.

Alpha constant has stimulated laboratory tests to improve the precision of measurements of the constancy of the fine structure constant [8] [9] [10]. The value of α is approximately equal to 1/137, and its exact value according to CODATA 2014 [11] is 0.0072973525664. It can be determined with a precision better than a few parts in 10^{-7} using four independent ways: the AC Josephson effect, the quantized Hall effect, the muonium hyperfine structure, and the electron anomalous magnetic moment. A determination of alpha based on an improved theoretical calculation and the Penning traps measurement of the electron anomalous magnetic moments reached a precision exceeding 10^{-8} [12]. Currently, the value of α with the smallest uncertainty was obtained from the comparison of the theoretical expression and experimental value of the anomalous magnetic moment of the electron. Starting in the 1980s, a new and wholly different measurement approach using the quantum Hall effect (QHE) has caused excitement because the value of α obtained from it independently corroborates the value of α from the electron magnetic moment anomaly. The QHE value of α does not have as small uncertainty as the electron magnetic moment value but provides significant independent confirmation of that value.

Recently, evidence indicating cosmological variations and the fine structure constant may be drifting [13] [14] and has triggered much interest in theories that account for the drift in fundamental constants [14]-[20].

The quantity α, which is equal to the ratio v/c, where v is the velocity of the electron in the first circular Bohr orbit, and c is the speed of light in vacuum, appeared naturally in Sommerfeld's analysis and determined the size of the splitting or fine structure of the hydrogenic spectral lines. However, it has remained

enigmatic for over 100 years and is considered to be fundamental and not derived. no compelling theory on its origin or a mechanism that explains its numerical value or the range of its domain exists, and it was a key unsolved physical problem for many physicists, such as Max Born, Richard Feynman [21], and Wolfgang Pauli [22], who wrote in 1948: "The theoretical interpretation of [the fine structure constant's] numerical value is one of the most important unsolved problems of atomic physics." Richard Feynmann called it a "magic number" and its value "one of the greatest damn mysteries of physics." [23] Some modern theories, such as String theory or anti-de Sitter/conformal field theory (AdS/CFT), propose mechanisms on how this constant emerges from more fundamental objects but fail to predict its value.

Therefore, despite attempts that have continued to date to find a mathematical basis for this dimensionless constant, no numerological explanation has yet been accepted by the community. In this work, a natural and compelling answer to the longstanding mystery of the meaning of α is proposed. The arguments presented below are based on the vortex model for the electron and the Hydrogen atom. A brief description of the electron structure is presented, and the essence and origin of the fine structure constant are derived. The proposed idea in this paper is that the electron is an irrotational vortex of frictionless superfluid space with concentric streamlines made up by massless Higgs particles, which acquire mass when they travel around the vortex center. According to this model, every elementary particle is made up of flux massless photons, which flow in a helix at the speed of light *c*. The fine structure constant is the ratio between the rotational velocity of the boundaries and the center of the electron and Hydrogen vortex. Therefore, α is dimensionless, and it has the same value on each discrete cosmological scale of nature.

2. The Structure of the Electron

Despite the impressive successes and impeccable mathematical tools in the application of quantum mechanics to many modern fields (such as semiconductors and superconductivity), the physicists cannot still advance a physical theory to answer the simple question "What is an electron?" The structure of the electron is not known, and the questions about the nature, shape, and size of the electron rarely have any place in modern physics. According to the quantum mechanics, the electron has no known substructure [24] [24], and the current understanding is that the electron is a point particle with a point charge and no spatial extent [26]. Attempts to model the electron as a non-point particle have been described as ill-conceived and counter-pedagogic. Therefore, the radius of the electron is a challenging problem of modern theoretical physics, and the admission of the hypothesis of a finite radius of the electron is incompatible to the premises of the theory of relativity.

On the other hand, a point-like electron (zero radius) generates serious mathematical difficulties due to the self-energy of the electron approaching infinity [27].

If the electron has a mass and no size is to say that it has infinite density. However, infinities of any quality rarely have a place in the real world. Nevertheless, it is useful to define a length that characterizes electron interactions in atomic-scale problems. Thus, it is fair to suggest that an electron does have a non-zero size, even though it is exceedingly small and irrelevant in most considerations.

The classical understanding of an electron accepts that the electron has an extension, and the vortex shape can explain the angular momentum (spin), magnetic moment, and the internal oscillation. In 1861, James Clerk Maxwell, described the electron as a vortex. He attempted to explain the magnetic field in terms of a sea of such excessively small whirlpools. In his paper "On Physical Lines of Force" [28]. He uses such a concept to explain magnetism on the basis that these vortices are aligned solenoidally with their rotation axes tracing out magnetic lines of force. He described the vortex lines as "lines of force" that are sometimes called "flux," meaning "flow lines." This vortex model helped him derive his famous Maxwell equations by which he unified magnetism and static electricity into a single theory of forces.

In 1928, when Paul Dirac presented the wave function of the electron (the "Dirac equation"), it became obvious that there must be not only an internal oscillation but also some internal motion at the speed of light.

When Erwin Schrödinger found it as a result of the Dirac equation, he gave the phenomenon the German name "Zitterbewegung," meaning a type of poorly defined oscillation. Jehle spent a large part of his life developing an electron theory and elementary particles based on quantized magnetic flux loops, spinning at the Zitterbewegung frequency [29] [30] [31] [32].

Both Dirac and Jehle theories rely on a physical relationship between flux and charge. The question of the relation between electric and magnetic properties is fundamental to electrodynamics. One expects a relationship because a moving charge produces magnetic flux.

A spinning system along an axis with an angular momentum has a torque when the force is directed toward the center of gravity known as Coriolis effect.

The flow to the center of the vortex due to Coriolis effect becomes vortex tube, which is always composed of the same virtual particles that rotate at the speed of light. Because it remains *unbroken*, it has a ring-like structure.

Maxwell assumed that every magnetic tube of force was a vortex with an axis of rotation coinciding with the direction of the force. Several properties have been mathematically proved for a perfect frictionless fluid [33].

In previous paper, a new theory was proposed in which the electron has a structure and a shape [34].

The vortex shape of the electron provides the correct relationship between the parameters of the electron, such as its mass, density volume, time, constant angular momentum (spin), electric charge, and magnetic moment. The electron as an irrotational circular vortex of frictionless superfluid space with concentric streamlines that was created from the primordial vacuum during the Big Bang.

The superfluid accommodates rotation by forming a lattice of quantized vortices in which the vortex core (typically singular) breaks the topological constraint against rotational motion.

In such a vortex, the magnitude of the vorticity in a vortex tube proportionally increases as the vortex line stretched. Consider a very thin vortex tube round the vortex line, so thin that the vorticity is practically constant over its width. As the vortex tube stretches, the cross-sectional area decreases by the same factor, so the vorticity must increase proportionally for the flux across the cross section to remain constant.

Therefore, the rate of rotation of the fluid is the greatest at the center and progressively decreases with distance from the center until there is no gradient pressure in the boundaries of the vortex where the flow will be laminar and the friction is null. If the speed of the space circulation reaches the limiting speed of light c in the absolute vacuum, and the velocity-field gradient around the center of the vortex becomes the postulated limiting angular rotation ω, space breaks down, creating a spherical void, which is defined as a field-less, energy-less and space-less volume of nothingness at the vortex center.

These maxima occur at the point where the centrifugal force and radial force are equalized, the inflowing medium and the free surface dip sharply, the inflowing medium turns at 90 degrees near the axis line with depth and velocity inversely proportional to r^2 to form a concave paraboloid.

3. Relation between Electron Vortex and the Fine Structure Constant

The fine structure constant α was first interpreted as the ratio of the velocity of the electron in the first circular orbit of the relativistic Bohr atom to the speed of light in the vacuum [35]. The fine structure constant was proposed by Sommerfeld as the ratio of the speed of the electron v in the ground state of Bohr's Hydrogen atom model to the speed of light c [36]:

$$\alpha = v/c = e^2 2/\hbar c \,,$$

where e is the charge of the electron. However, why this ratio is constant is not known. The value of constant α is a dimensionless quantity, which indicates that this value is an expression of the ratio between two quantities that have the same units.

The irrotational vortex structure is universal, which can found in the micro realm, such as the electron structure and Hydrogen atom structure, as well as in the macro, such as in the spiral galaxies. In fluid dynamics, the irrotational vortex dynamics has two different rotational speeds at two different radii; in the electron and the Hydrogen atom, the internal one, where the rotation velocity is at the speed of light and the second radius from the center to the boundaries. The rotational speed in the boundaries is calculated and the ratio between the external rotational speed and the center speed is derived. The constant alpha related to different constant ratios present in the irrotational vortex is the ratio

between the rotational speed on the boundaries of the vortex and speed of light in the center. Since the angular velocity ω in the vortex is constant according to the equation

$$\omega = cr_c$$

Thus, the ratio of the radius of the core of the vortex to the radius from the boundaries to the center is constant.

In hydrodynamics, the velocity of the fluid element instantaneously passing through a given point in space in the vortex with radius r is constant in time; therefore, the circulation or the vorticity Γ of the vortex is

$$\Gamma = 2\pi r_e c$$

where $2\pi r_e$ is the circumference of the electron vortex.

Since Γm is the conserved momentum, $2\pi rcm$ is constant, which corresponds to the Planck constant. Therefore, the Planck constant can be expressed as

$$h = 2\pi rcm$$

Knowing the mass of the electron, then the radius of the eye of the vortex, which rotates at speed of light is calculated as

$$r = h/2\pi m_o c = 3.86 \times 10^{-13}\,\text{m}$$

where m_o is the rest mass of an electron = $9.10938356 \times 10^{-31}$ kg, and $h = 6.61997943364 \times 10^{-34}$ kg·m²·s⁻¹, and c is the speed of light, $c = 3 \times 10^8$ m·s⁻¹.

The radius on the boundaries of the electron vortex can be calculated if the rotational speed is known.

In the electron vortex model, the electric charge is an expression of the volume flow rate of vacuum flux from the vacuum to the center of the electron vortex. The electric force is the force needed to move the flow from the periphery to the center. The force acting on the two charges is expressed by Coulomb's law expressed as

$$F = e^2 / 4\pi r_e^2 \varepsilon_0$$

In the vortex, this force is equivalent to the centripetal force $m_o v^2/r$, where m is the rest mass of the electron, v is the rotational speed at distance r from the center, and r is the radius of the electron. Therefore,

$$F = m_o v^2 / r = e^2 / 4\pi r^2 \varepsilon_0 \,.$$

Based on this equation, the rotational speed of the vortex is

$$v = e^2 / 4\pi r m_o v \varepsilon_0$$

Since $2\pi rm_o v$ is the conserved momentum and constant, the rotational speed of the vortex is equal to Planck constant h.

Therefore,

$$v = e^2 / 2h\varepsilon_0 = 2.1876913 \times 10^6\,\text{m/s}$$

where ε_0 is the electric permittivity ($8.854187817... \times 10^{-12}$ F·m⁻¹), e is the electric charge ($1.602176634 \times 10^{-19}$ C), and h is the Planck constant ($6.62607004 \times 10^{-34}$ m²·kg/s).

The ratio between the rotational speed on the boundaries and in the center of the vortex is equivalent to v/c, which can be calculated as

$$2.1892212626 \times 10^6 / 3 \times 10^8 = 0.007292304333 = 1/137.13$$

and is the same value of α.

From the rotational velocity, the radius of the electron r_e vortex in the boundaries can be calculated as

$$r_e = h/2\pi vm = 5.2895948 \times 10^{-11} \, \text{m}.$$

The ratio between the radius at the boundaries of the electron r_e and the radius of the vortex the at the center r_c is

$$r_e/r_c = 3.86 \times 10^{-13} / 5.2895948 \times 10^{-11} = 0.007297345347 = 1/137.036,$$

which is also has the same value of α, eliminating the uncertainty of the elliptic radius.

4. Relation between Hydrogen Model and the Fine Structure Constant

The fine structure constant in the formula for the energy levels of the Hydrogen atom was first given by Sommerfeld. Bohr's model of the atom postulated that the electrons of an atom moved about its nucleus in circular orbits, or as later suggested by Arnold Summerfeld (1868-1951), in elliptical orbits, each with a certain "allowed" energy and relativistic dependence of mass on velocity.

According to quantum mechanics, an electron orbital is the position of the electrons around the nucleus and is determined as the volume of space in which the electron can be found with a 95% probability. Each orbital has a specific energy. The position (the probability amplitude) of the electron is defined by its coordinates in space, which is indicated by $\psi(x, y, z)$ in Cartesian coordinates. ψ cannot be measured directly but is a mathematical tool.

The clouds of probabilities are known as shells. Each shell has sublevels and subshells. The numbers of electrons that can occupy each shell and each subshell arise from the equations of quantum mechanics, in particular, the Pauli exclusion principle, which states that no two electrons in the same atom can have the same values of the four quantum numbers. However, no theory explains the nature or essence of the shells, sublevels, and subshells. Thus, the vortex model was applied to the shell structure, which was discussed in detail in other papers.

In this article, the Hydrogen shell structure is considered and treated as a vortex, where the proton in the center of the vortex and the electron located on one of the spiral lines of the vortex. Therefore, the electron orbital rotation is not free but is guided by the vortex rotation that produces the magnetic field of the atom. The rotational speed of the proton vortex center is the speed of light as it is in the center of the electron vortex. The internal radius of the proton is calculated as

$$r_c = h/2\pi cm_p = 2.103104894 \times 10^{-16} \, \text{m}$$

where m_p is the mass of the proton ($1.672621898 \times 10^{-27}$ kg). The electron charge force is the force of attraction found on the boundaries of the electron that interact with the attractive force of the Hydrogen vortex, which is equal to the attractive gravitation force of the Hydrogen vortex. Therefore, the rotational velocity of the electron around the proton is

$$v_e = Ze^2 \big/ 2h\varepsilon_0 = 2.1876913 \times 10^6 \text{ m/s}$$

where Z is the Hydrogen atomic number, 1, and the ratio between the electron orbital velocity around the proton and vortex velocity at the core of the proton is $v_e/c = 2.1876913 \times 10^6 / 2.9979245 \times 10^8 = 0.00729736383 = 1/137.035$ which is equal to α constant. Since the vorticity of the vortex is constant, the angular rotation $\omega = cr_c$ is constant. The angular rotation in the center of the proton vortex $\omega = cr_c = 6.3082019259 \times 10^{-8}$ has the same angular rotation of the electron around the proton.

If the velocity of the electron around the proton is 2.1876913×10^6 m/s, then the distance of the electron from the proton r_p is

$$r_p = \omega/v = 6.3082019259 \times 10^{-8} \big/ 2.1876913 \times 10^6 = 2.8834972859 \times 10^{-14} \text{ m} .$$

The ratio between the radius of the proton vortex core and in the radius where the electron is located can be calculated:

$$r_c \big/ r_p = 2.103104894 \times 10^{-16} \big/ 2.8834972859 \times 10^{-14} = 0.007292304332 = 1/137.13 ,$$

which is equal to fine structure constant α.

Therefore, the Hydrogen has a vortex structure similar to the electron structure with a different radius. This does not necessarily indicate the radius of the proton but the distance of the electron from the center of the proton in the Hydrogen atom.

5. Conclusions

A new theory that describes the origin of the fine structure constant is presented, based on the structures of the electron and the Hydrogen atom. Both are considered as irrotational superfluid vortices with a permanent flow pattern and differential rotational velocity at the core of the vortex relative to its boundaries. Previous article [36] described the nature and the origin of Constant G based on superfluid vortex theory.

The radius of the electron vortex core, which rotates at speed of light was calculated to have the same Coulomb radius value. The tangential velocity was calculated based on the centripetal force on the boundaries of the electron, which is equal to electric force between two charges according to Coulomb flux of force. Once the tangential velocity was determined, the radius of the electron from the center to the boundaries was calculated.

The same vortex model was applied to the Hydrogen structure. The orbital velocity of the electron around the proton and the radius between the electron and proton was calculated. The fine structure constant is proportional to the ra-

tio between the tangential velocity of the electron around the proton and the speed of light at the center of the proton.

Consequently, we showed that the fine structure constant considering the electron as a vortex in which the core is rotating at the speed of light, the ratio between the core radius of the electron vortex and the radius to the boundaries has the value of the fine structure constant. Furthermore, considering the Hydrogen structure as an irrotational vortex, and the electron rotates around the proton with constant angular rotation, the ratio between the radius of the proton at its core and the distance of the electron orbital gives the same value of the fine structure constant. We conclude that the fine structure constant may not be a fundamental constant but is an expression of the constancy of the ratio of the tangential velocity of irrotational vortices to the core velocity and the ratio of the core radius to the vortex radius.

Acknowledgements

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Uzan, J.-P. (2003) The Fundamental Constants and Their Variation: Observational and Theoretical Status. *Reviews of Modern Physics*, **75**, 403-455. https://doi.org/10.1103/RevModPhys.75.403

[2] Carroll, S.M. (2010) The Fine Structure Constant Is Probably Constant.

[3] Michelson, A.A. and Morley, E.W. (1887) Phil Mag. 24. *American Journal of Science*, **34**, 427. https://doi.org/10.2475/ajs.s3-34.204.427

[4] Sommerfeld, A. (1923) Atomic Structure and Spectral Lines. Methuen, London, 118.

[5] Kragh, H.N. (2012) Bohr and the Quantum Atom. Oxford University Press, Oxford. https://doi.org/10.1093/acprof:oso/9780199654987.001.0001

[6] Sommerfeld, A. (1940) Zur Feinstruktur der Wasserstofflinien. Geschichte und gegenwärtiger Stand der Theorie. *Naturwissenschaften*, **28**, 417-423. (In German) https://doi.org/10.1007/BF01490583

[7] Bouchendira, R., Cladé, P., Guellati-Khélifa, S., Nez, F. and Biraben, F. (2011) New Determination of the Fine-Structure Constant and Test of the Quantum Electrodynamics. *Physical Review Letters*, **106**, Article ID: 080801. https://doi.org/10.1103/PhysRevLett.106.080801

[8] Prestage, J.D., Tjoelker, R.L. and Maleki, L. (1995) Atomic Clocks and Variations of the Fine Structure Constant. *Physical Review Letters*, **74**, 3511-3514. https://doi.org/10.1103/PhysRevLett.74.3511

[9] Damour, T. and Dyson, F. (1996) The Oklo Bound on the Time Variation of the Fine-Structure Constant Revisited. *Nuclear Physics B*, **480**, 37-54. https://doi.org/10.1016/S0550-3213(96)00467-1

[10] Salomon, C., Dimarcq, N., Abgrall, M., *et al.* (2001) Cold Atoms in Space and Atomic Clocks. Comptes rendus de l'Académie des Sciences, Paris, Vol. IV, 1-17.

[11] Mohr, P.J., Taylor, B.N. and Newell, D.B. (2015) The CODATA Recommended Values of the Fundamental Physical Constants (Web Version 7.0). http://physics.nist.gov/constants https://doi.org/10.6028/NIST.SP.961r2015

[12] Gabrielse, G. (2009) Why Is Sideband Mass Spectrometry Possible with Ions in a Penning TRAP? *Physical Review Letters*, **102**, Article ID: 172501. https://doi.org/10.1103/PhysRevLett.102.172501

[13] Webb, J.K., Flambaum, V.V., Churchill, C.W., Drinkwater, M.J. and Barrow, J.D. (1999) Search for Time Variation of the Fine Structure Constant. *Physical Review Letters*, **82**, 884-887. https://doi.org/10.1103/PhysRevLett.82.884

[14] Webb, J.K., Murphy, M.T., Flambaum, V.V., Dzuba, V.A., Barrow, J.D., Churchill, C.W., *et al.* (2001) Further Evidence for Cosmological Evolution of the Fine Structure Constant. *Physical Review Letters*, **87**, Article ID: 091301. https://doi.org/10.1103/PhysRevLett.87.091301

[15] Bekenstein, J.D. (2002) Physical Review. Part D. 66.

[16] Damour, T., Piazza, F. and Veneziano, G. (2002) Runaway Dilaton and Equivalence Principle Violations. *Physical Review Letters*, **89**, Article ID: 081601. https://doi.org/10.1103/PhysRevLett.89.081601

[17] Davies, P.C., Davis, T.M. and Lineweaver, C.H. (2002) Black Holes Constrain Varying Constants. *Nature*, **418**, 602-603. https://doi.org/10.1038/418602a

[18] Forgacs, P. and Horvath, Z. (1979) On a Static Solution of the Einstein-Yang-Mills System in Six Dimensions. *General Relativity and Gravitation*, **10**, 931-940. https://doi.org/10.1007/BF00756757

[19] Forgacs, P. and Horvath, Z. (1979) On the Influence of Extra Dimensions on the Homogeneous Isotropic Universe. *General Relativity and Gravitation*, **11**, 205-216. https://doi.org/10.1007/BF00762129

[20] Magueijo, J. to be published in Reports in Progress of Physics.

[21] Resonance (1999) 14(4).

[22] Resonance (2011) 16(9).

[23] Eichten, E.J., Lane, K.D. and Peskin, M.E. (1983) New Tests for Quark and Lepton Substructure. *Physical Review Letters*, **50**, 811-814. https://doi.org/10.1103/PhysRevLett.50.811

[24] Gabrielse, G., Hanneke, D., Kinoshita, T., Nio, M. and Odom, B. (2006) New Determination of the Fine Structure Constant from the Electron Value and QED. *Physical Review Letters*, **97**, Article ID: 030802. https://doi.org/10.1103/PhysRevLett.97.030802

[25] Curtis, L.J. (2003) Atomic Structure and Lifetimes: A Conceptual Approach. Cambridge University Press, Cambridge, 74. https://doi.org/10.1017/CBO9780511755552

[26] Shpolsky, E. (1951) Atomic Physics. 2nd Edition.

[27] Clerk-Maxwell, J. (1861) On Physical Lines of Force. *Philosophical Magazine*, **21**, 338-348. http://vacuum-physics.com/Maxwell/maxwell_oplf.pdf

[28] Jehle, H. (1971) Relationship of Flux Quantization to Charge Quantization and the Electromagnetic Coupling Constant. *Physical Review D*, **3**, 306-345. https://doi.org/10.1103/PhysRevD.3.306

[29] Jehle, H. (1972) Flux Quantization and Particle Physics. *Physical Review D*, **6**, 441-457. https://doi.org/10.1103/PhysRevD.6.441

[30] Jehle, H. (1975) Flux Quantization and Fractional Charges of Quarks. *Physical Review D*, **11**, 2147-2177. https://doi.org/10.1103/PhysRevD.11.2147

[31] Jehle, H. (1977) Electron-Muon Puzzle and the Electromagnetic Coupling Constant. *Physical Review D*, **15**, 3727-3759. https://doi.org/10.1103/PhysRevD.15.3727

[32] Maxwell, J.C. (2010) On Physical Lines of Force. *Philosophical Magazine*, **90**, 11-23. https://doi.org/10.1080/14786431003659180

[33] Butto, N. (2020) Electron Shape and Structure: A New Vortex Theory. *Journal of High Energy Physics, Gravitation and Cosmology*, **6**, 340-352. https://doi.org/10.4236/jhepgc.2020.63027

[34] Introduction to the Constants for Nonexperts—Current Advances: The Fine-Structure Constant and Quantum Hall Effect. NIST Reference on Constants, Units and Uncertainty. NIST.

[35] Sommerfeld, A. (1916) On the Quantum Theory of Spectral Lines. *Annals of Physics*, **51**, 125-167.

[36] Butto, N. (2020) New Mechanism and Analytical Formula for Understanding the Gravity Constant G. *Journal of High Energy Physics, Gravitation and Cosmology*, **6**, 357. https://doi.org/10.4236/jhepgc.2020.63029